A three-layer model of the global overturning circulation

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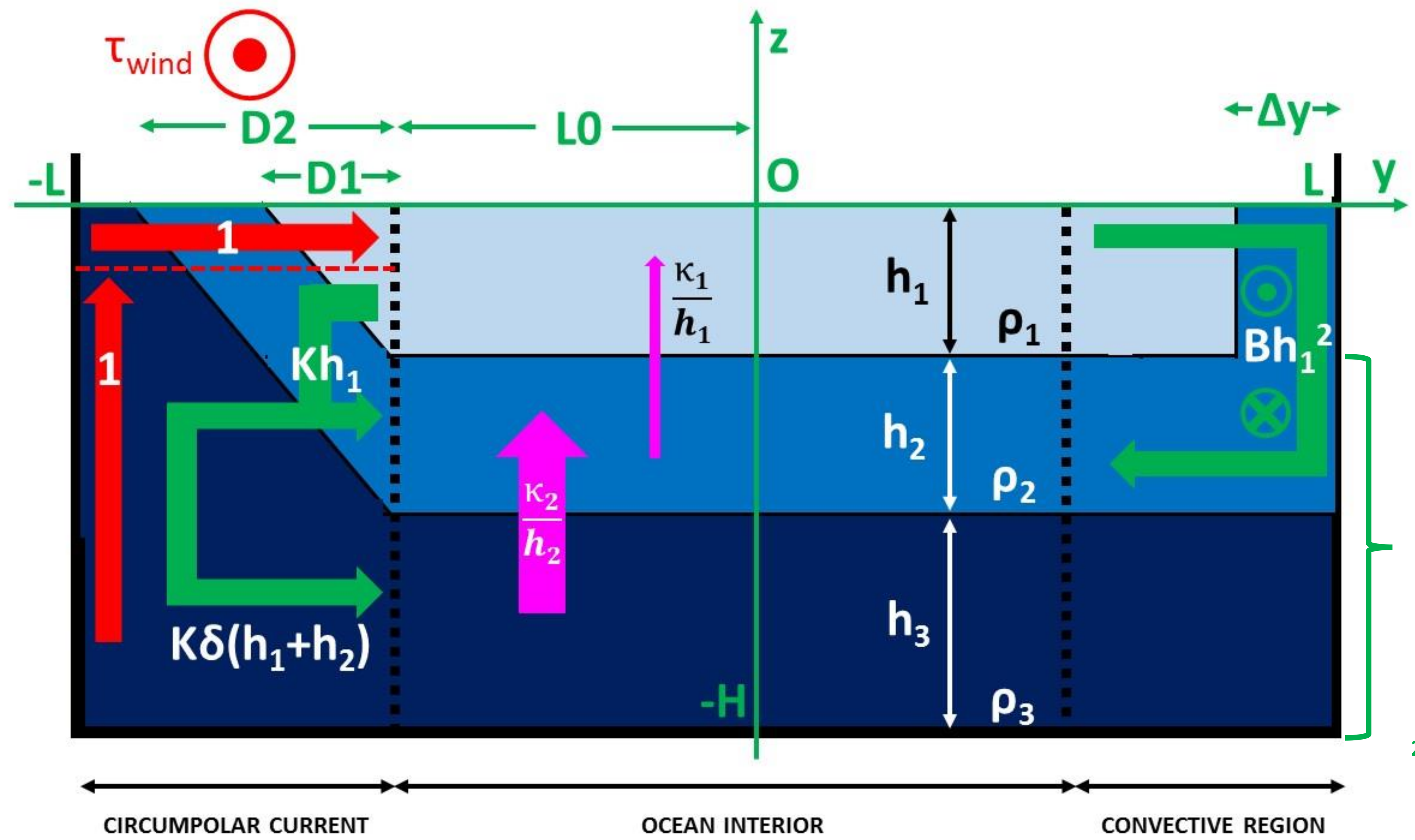
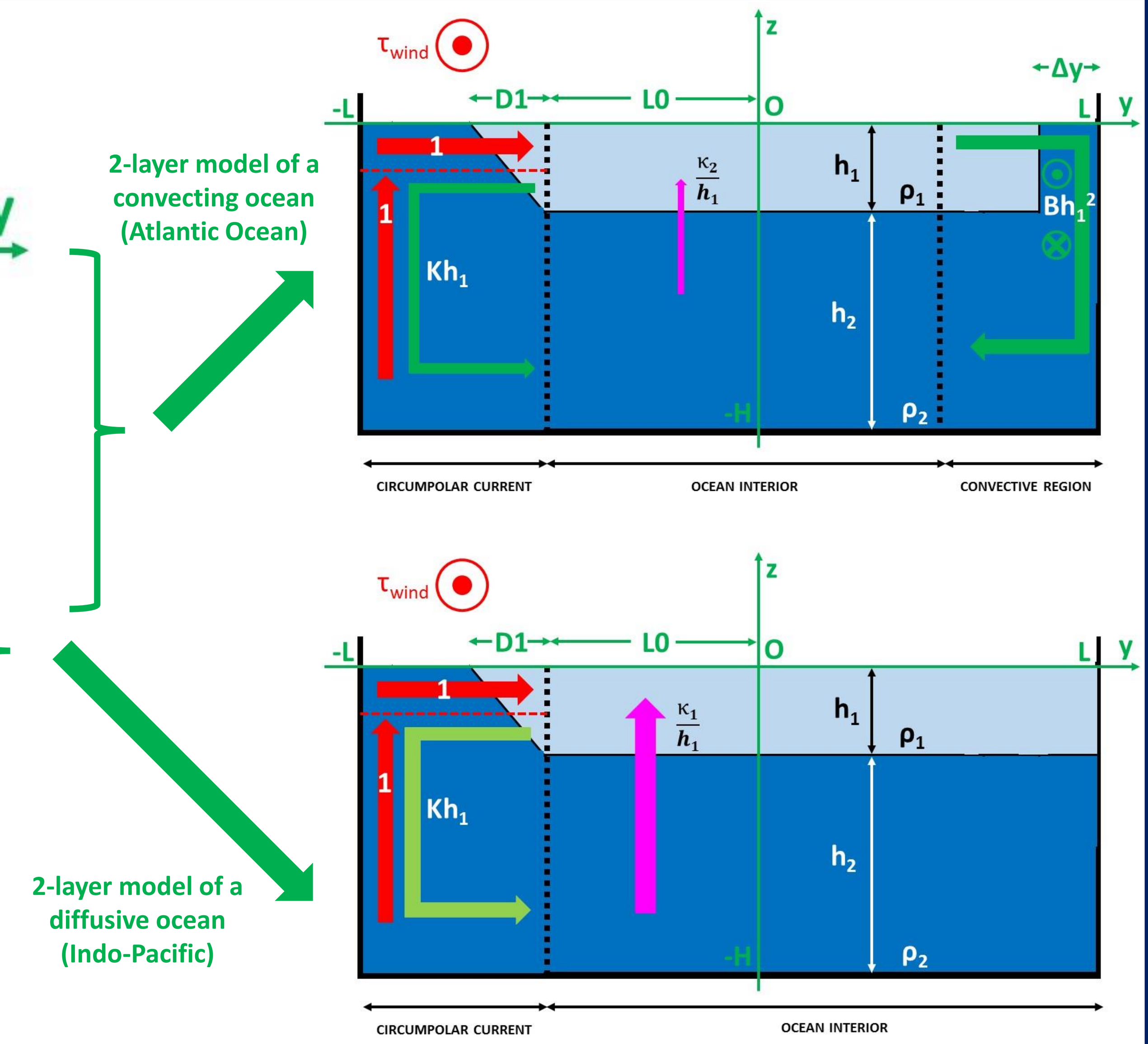
1. Model





The circumpolar channel extends from the south Pole (y=-L) to (y=-L0). We assume that the mean isopycnals have a constant slope and that the westerlies exert a constant stress τ at the surface. By using a transformed mean Eulerian framework and Gent & McWilliam's closure for the eddy fluxes, the residual streamfunction from layer i to layer (i+1) can be written:

$$\psi_i^\dagger = \underbrace{-\frac{\tau}{\rho_0 f}}_{\text{wind-induced}} + \underbrace{K_{\text{eddy}}\alpha_i}_{\text{eddy-driven}}$$

Where the slopes are given by: $\alpha_1 = h_1 D_1^{-1}$

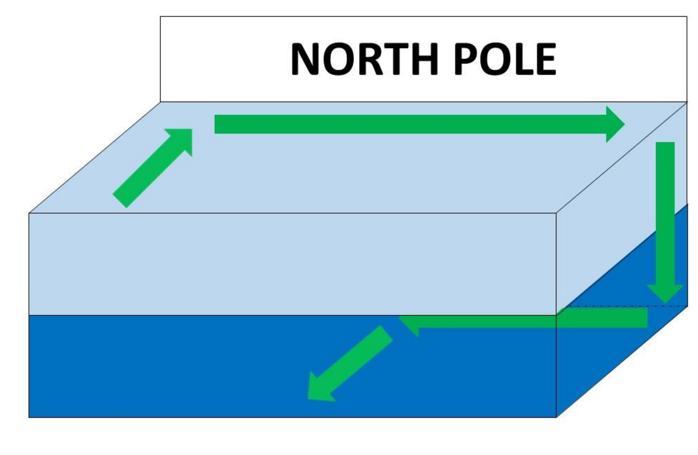
$$\alpha_2 = (h_1+h_2)D_2^{-1}$$

We assume that the mean isopycnals are flat in the ocean interior. We discretize the advection-diffusion equation for buoyancy to relate the residual circulation from the Southern Ocean and the North Pole circulation to the vertical velocity in the ocean interior:

$$\frac{\varepsilon\psi_{\text{polar}} - \psi^+(-L_1)}{L+L_0}\frac{b_{12}}{h_1} \approx \kappa_{\text{con}}\frac{b_{12}}{h_1^2}$$

The flow follows the polar circulation depicted on the right, induced by the atmospheric buoyancy gradient between the mid-latitudes and the North Pole. We compute the zonal flow from the thermal wind relation, which gives us the streamfunction associated to this transport:

$$\psi_{\text{polar}} = -\int_{-h_1}^{0} u_{\text{polar}}\,dz \approx \frac{h_1^2}{f}\frac{\Delta b}{\Delta y}$$



2. Mass conservation

$$T\dot{h}_1 = \underbrace{1}_{\text{wind}} + \underbrace{\frac{\kappa_1}{h_1}}_{\text{mixing}} - \underbrace{Kh_1}_{\text{eddies}} - \underbrace{Bh_1^2}_{\text{convection}}$$

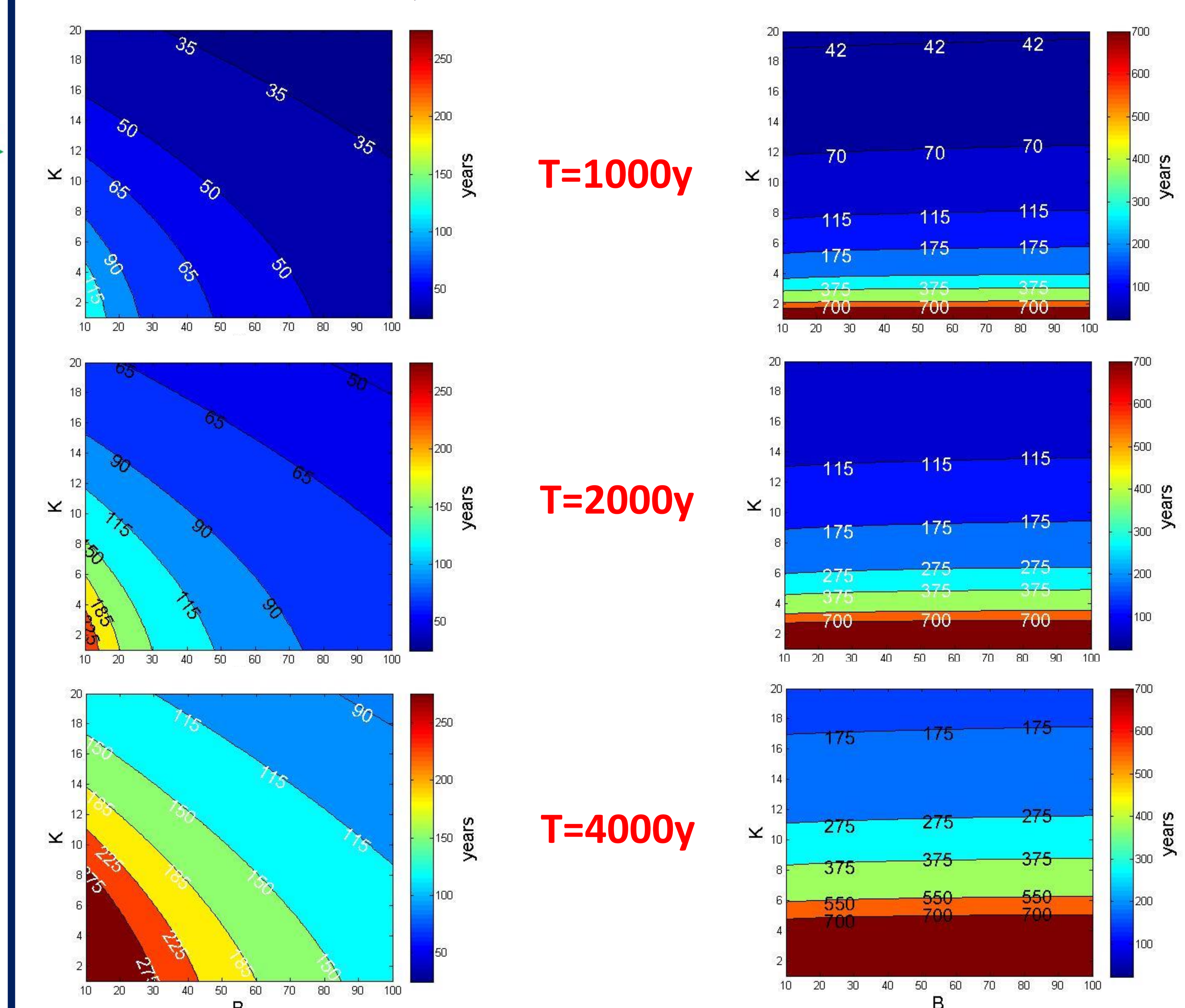
$$T\dot{h}_2 = \underbrace{Kh_1}_{\text{eddies}} + \underbrace{Bh_1^2}_{\text{convection}} + \underbrace{\frac{\kappa_2}{h_2}}_{\text{mixing}} - \underbrace{\frac{\kappa_1}{h_1}}_{\text{mixing}} - \underbrace{K\delta(h_1+h_2)}_{\text{eddies}}$$

$$T\dot{h}_3 = \underbrace{K\delta(h_1+h_2)}_{\text{eddies}} - \underbrace{1}_{\text{wind}} - \underbrace{\frac{\kappa_2}{h_2}}_{\text{mixing}}$$

The three equations on the left describe the evolution of the thicknesses of each layer. This mass budget can be simply computed by subtracting what comes out of the layer i to what comes in the layer i. From these equations, we can estimate the typical thicknesses relaxation times, when the system is perturbed from its equilibrium state. These dynamic timescales can be expressed as functions of the parameters (K,B,T), allowing us to address a broad range of climate change scenarios.

$$T_{h1,\text{GOC}} = \frac{T}{2K + 3Bh_{1,s} - h_{1,s}^{-1}}$$

$$T_{h2,\text{GOC}} = \frac{T}{K\delta(h_{1,s}h_{2,s}^{-1}+2) - 1}$$



3. Heat conservation

$$T\frac{d(h_1\theta_1)}{dt} = \underbrace{\theta_3}_{\text{wind}} + \underbrace{\frac{\kappa_1\theta_2}{h_1}}_{\text{mixing}} - \underbrace{(Kh_1 + Bh_1^2)\theta_1}_{\text{eddies+convection}}$$

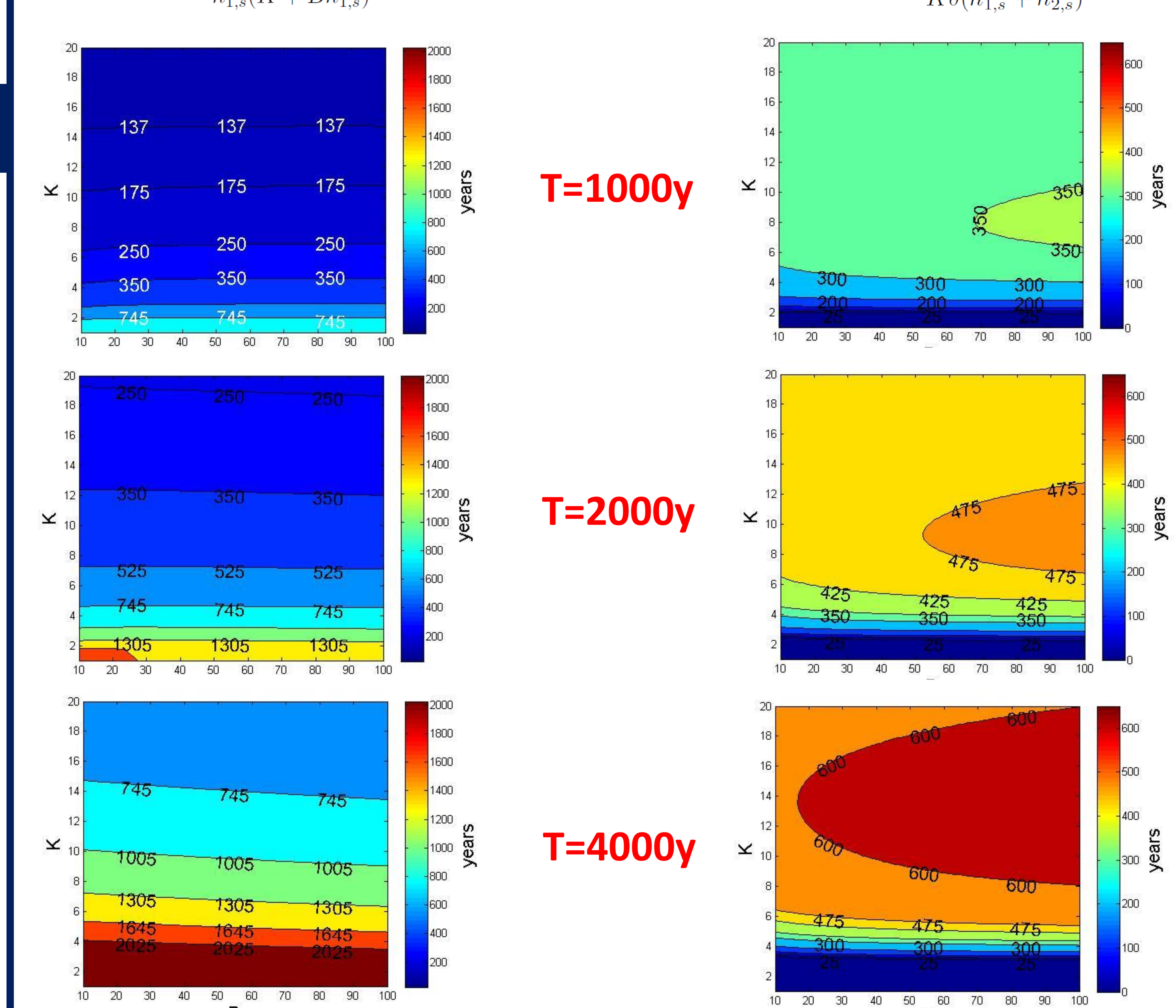
$$T\frac{d(h_2\theta_2)}{dt} = \underbrace{(Kh_1 + Bh_1^2)\theta_1}_{\text{eddies+convection}} + \underbrace{\frac{\kappa_2\theta_3}{h_2}}_{\text{mixing}} - \underbrace{[\frac{\kappa_1}{h_1} + K\delta(h_1+h_2)]\theta_2}_{\text{mixing+eddies}}$$

$$T\frac{d(h_3\theta_3)}{dt} = \underbrace{K\delta(h_1+h_2)\theta_2}_{\text{eddies}} - \underbrace{(1+\frac{\kappa_2}{h_2})\theta_3}_{\text{wind+mixing}}$$

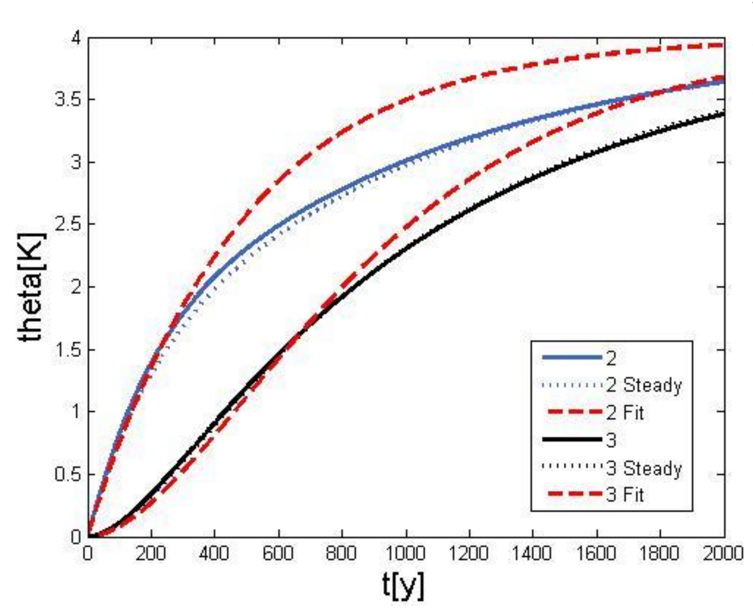
The three equations on the left describe the evolution of the heat content of each layer. This heat budget can be simply computed by considering the advective heat fluxes in the system. From these equations, we can estimate the typical adjustment time T(i→j) of a layer i when a layer j is heated/cooled. If we assume the thicknesses to be steady, these thermodynamic timescales can be expressed as functions of the parameters (K,B,T), allowing us to address a broad range of climate change scenarios.

$$T_{2\to1} = \frac{h_{2,s}T}{h_{1,s}(K + Bh_{1,s})}$$

$$T_{3\to2} = \frac{(1 - h_{1,s} - h_{2,s})T}{K\delta(h_{1,s} + h_{2,s})}$$



4. Is heat passive or active?



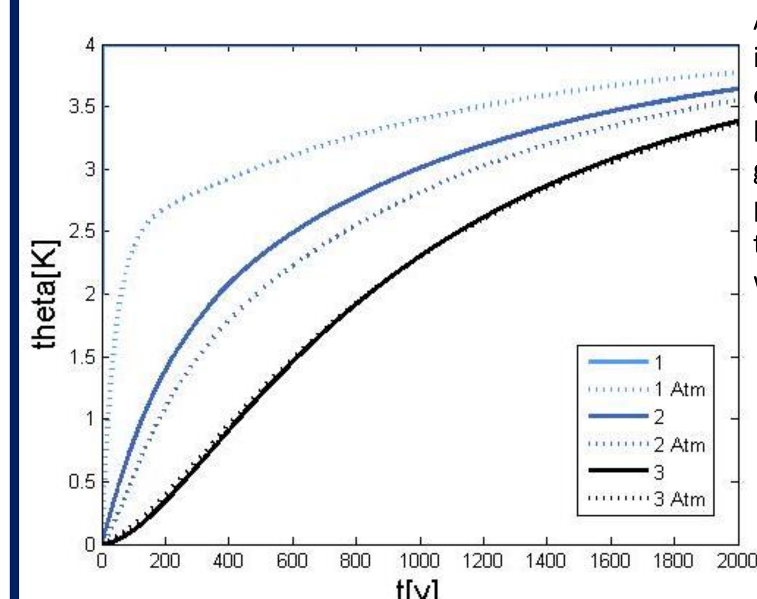
To test the passive/active nature of the ocean heat uptake in this model, we run an idealized global warming scenario. The scenario includes an instantaneous 4°C warming of the upper layer, a one third increase in the wind stress and the eddy diffusivity, and a one third decrease in the mid-latitude/pole atmospheric buoyancy gradient because of polar amplification:

$$\theta_1(t) = \Delta\theta\cdot\mathcal{H}(t) \qquad \Psi_{\text{cc}}(t) = \Psi(1+\frac{\mathcal{H}(t)}{3})$$

$$\Delta b_{\text{cc}}(t) = \Delta b(1-\frac{\mathcal{H}(t)}{3}) \qquad K_{\text{cc}}(t) = K_{\text{eddy}}(1+\frac{\mathcal{H}(t)}{3})$$

On the left, we can see that assuming steady thicknesses has very little effect on the growth of temperature perturbations, leading to think that the heat can be approximated as passive for this scenario. The two exponential fits based on the adjustment timescales derived in part 3 approximate the signal well for the first few centuries of the scenario.

5. Effect of an idealized atmosphere



A simple way to model ocean-atmosphere heat fluxes is to introduce a single linear exchange coefficient λexc. In the heat conservation of part 3, it adds a term $\lambda_{\text{exc}}S(\theta_{\text{atm}} - \theta_i)$ to the right hand side of each equation. We are now in position to refine the global warming scenario of part 4 by introducing the heat perturbation in the atmosphere rather than in the upper layer of the ocean. The upper layer of the ocean adjusts to the atmospheric warming with the following timescale:

$$T_{1\to\text{atm}} = \frac{Th_{1,s}}{\lambda_1}$$

The two other layers then uptake heat, partly from the atmosphere, but mostly because of the advection of heat within the ocean. From this, we see that the atmosphere only plays a role on short timescales, i.e. a few decades at most, if we assume the global exchange coefficient to be order $1Wm^{-2}K^{-1}$.

Parameters

Wind-induced streamfunction (ref)

$$\Psi \stackrel{\text{def}}{=} (\frac{\tau}{\rho_0 f})(y=-L_0)$$

Dimensionless eddy diffusivity

$$K \stackrel{\text{def}}{=} \frac{K_{\text{eddy}}}{\Psi}\frac{H}{D_1}$$

Dimensionless diapycnal mixing 12

$$\kappa_1 \stackrel{\text{def}}{=} \frac{\kappa_{\text{con}}}{\Psi}\frac{L+L_0}{H}$$

Dimensionless diapycnal mixing 23

$$\kappa_2 \stackrel{\text{def}}{=} \frac{\kappa_{\text{dif}}}{\Psi}\frac{L+L_0}{H}$$

Dimensionless buoyancy gradient

$$B \stackrel{\text{def}}{=} \frac{H^2}{f\Psi}\frac{\Delta b}{\Delta y}$$

Isopycnal's outcrop ratio

$$\delta \stackrel{\text{def}}{=} \frac{D_1}{D_2}$$

Ekman transport's timescale

$$T \stackrel{\text{def}}{=} \frac{SH}{W\Psi}$$

Dimensionless heat exchange coef

$$\lambda_i \stackrel{\text{def}}{=} \frac{\lambda_{\text{exc}}S_i}{c_l\Psi W}$$

Parameter table

Parameter	Description	Value
a	Radius of the Earth	$6.4.10^3$km
b_1	Buoyancy of the upper layer	43.1cm.s^{-2}
b_2	Buoyancy of the deep layer	42.0cm.s^{-2}
b_3	Buoyancy of the abyssal layer	40.9cm.s^{-2}
c_l	Volumetric heat capacity of liquid water at constant pressure	$4.2.10^6$J.m^{-3}.K^{-1}
D_1	Mean Isopycnal outcrop - Northern edge of the channel	$5.6.10^2$km (5°)
D_2	Second Isopycnal outcrop - Northern edge of the channel	$8.3.10^2$km (7.5°)
f	Coriolis parameter at the Northern edge of the basin	$1.1.10^{-4}$s^{-1} (50°)
g	Gravity constant	9.8m.s^{-2}
H	Mean ocean depth	4.0km
K_{eddy}	Lateral eddy diffusivity in the channel	$1.0.10^3$m^2.s^{-1}
L	Quarter of the Earth's perimeter	$1.0.10^4$km (90°)
L_0	Northern edge of the channel - Equator	$5.6.10^3$km (50°)
S	Total surface of the ocean	$3.5.10^{14}$m
T_1	Temperature of the upper layer	20.0°C
T_2	Temperature of the deep layer	12.5°C
T_3	Temperature of the abyssal layer	5°C
V_1	Volume of upper layer	$1.1.10^{17}$m^3
V_2	Volume of deep layer	$4.4.10^{17}$m^3
V_3	Volume of abyssal layer	$8.4.10^{17}$m^3
$\frac{\Delta b}{\Delta y}$	Polar buoyancy gradient	$2.5.10^{-9}$s^{-2}
α	Thermal expansion coefficient	$1.5.10^{-4}$K^{-1}
ε	Fraction of the ocean open to the polar convective zone	1/8 (45°/360°)
κ_{con}	Diapycnal interior diffusivity for a convective ocean	$1.0.10^{-5}$m^2.s^{-1}
κ_{dif}	Diapycnal interior diffusivity for a diffusive ocean	$1.0.10^{-4}$m^2.s^{-1}
Ψ	Wind-induced streamfunction	0.90m^2.s^{-1}

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